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Sensitivity analysis for tire/road interface model

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Abstract

The aim of this work is to highlight the more influential parameters involved in a tire/road interface model, the Magic Formula (also called Pacejka model), commonly used in the automotive and aeronautical domain. The paper takes an interest in the basic function of Magic Formula, which is nonlinear and depends on six unknown parameters to be estimated. A sensitivity analysis based on the approach of Sobol is carried out. It is then emphasized that only two parameters over the six contribute significantly to the uncertainty of the output of this model.

Keywords: sensitivity analysis; analysis of variance; nonlinear model; tire/road interface

1. Main text

In the field of automotive and aircraft, the modeling of the tire/road interface plays a crucial role. Indeed, the tires are the only contact surface with the ground and the major part of the efforts applied to the vehicles pass through them. Thus, in the literature, there exist a lot of models describing the efforts and moments acting between tires and road (see for example Pacejka(2006)). In general, these models are complex, nonlinear and depend on parameters that have to be identified from measurement data. Very often, few measures are available because they are costly, especially in the aeronautical domain, and the parameters are then estimated with more or less precision, leading to uncertainty on the efforts that can be unacceptable. In fact, among all the model parameters, some will contribute very few to the variation of the efforts. However, others will contribute in a significant manner to the effort variation. In this case, some additional measurements can be required in order to reduce the uncertainty on the parameters and thus on the efforts. Consequently, the problem is to determine the parameters that can be fixed to a value of their uncertainty interval because they are very few influential on the output. The sensitivity analysis allows to solve this problem. Different approaches for testing the sensitivity analysis have been proposed in the literature (Cukier et al(1978), Sobol(1993), Saltelli(1999, 2000, 2002, 2004), Frey et al(2002)).
The aim of this paper is to determine the influential parameters in a tire/road interface model, the basic function of Magic Formula (Pacejka(2006)), commonly used in the automotive and aeronautical domains.
Among the Magic Formula expressions for the tire/road interface characteristics (forces and moments), only the lateral force expression is considered in this paper. It corresponds to the road reaction on the tire in the lateral direction, during cornering maneuver, for instance. In the following, the used data come from the aeronautical domain. Besides, the parameters are considered as independent random variables. Magic Formula is an empirical model. In this model, in the case of steady state pure lateral slip, the lateral force F_y is given by the following equation:

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$$F_y = \mu F_z \sin\left(C \operatorname{atan}\left(\frac{K}{\mu F_z C}(\alpha + S_h) - E\left(\frac{K}{\mu F_z C}(\alpha + S_h) - \operatorname{atan}\left(\frac{K}{\mu F_z C}(\alpha + S_h) \right)\right)\right)\right) + S_v \qquad (1)$$

where K is the cornering stiffness, F_z the vertical load, μ the friction coefficient and α the side slip angle, that is the angle between the longitudinal axis of the wheel and its direction of travel. The terms C, E, S_v and S_h are empirical parameters. The lateral force F_y represents the model output. The parameters α and F_z are assumed known and respectively fixed to 10° and 90kN. The parameters μ, C, K, E, S_h and S_v are the unknown, which should be estimated. They are all supposed to follow a uniform law in the respective intervals [0.5;2.5], [1;2], [-1165kN/rad; -179.02kN/rad], [-3; 0.5], [-0.0037;0.0037] and [-3322;3322]. These bounds come from a data base existing in the aeronautical domain. For each parameter, a sample size N=100000 is considered. The figure 1 represents the cumulative frequency of F_y and the distribution histogram of the values. The mean value of F_y is -85.24kN and its 95% confidence interval is [-164.3kN;-35.5kN], underlining an uncertainty of 128.8kN, that is 151%, on F_y. It is obvious that this uncertainty is too large. Thus, it is worth analyzing the contribution of each parameter to this uncertainty. The sensitivity indices are estimated by the Monte Carlo approach (Sobol(1993)). The results are summed up in the Table 1.

1st order indices	total indices
$S_\mu = 0.3583$	$S_{T\mu} = 0.5216$
$S_K = 0.4659$	$S_{TK} = 0.6253$
$S_E \approx 0$	$S_{TE} = 0.0138$
$S_C \approx 0$	$S_{TC} = 0.0044$
$S_{Sv} \approx 0$	$S_{TSv} = 0.0036$
$S_{Sh} \approx 0$	$S_{TSh} = 0.0002$

Table 1 – Sensitivity indices values

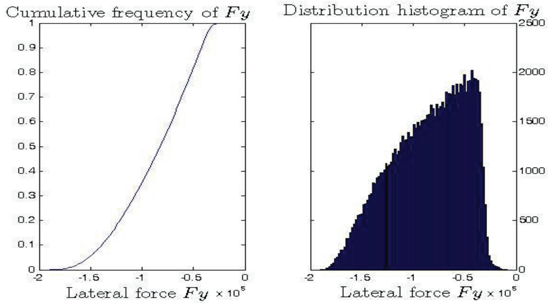

Fig. 1 – Cumulative frequency and distribution histogram of F_y

contribution on F_y can be considered as nill and thus, these parameters can be fixed to a value from their interval of variation without significant effect on the force F_y uncertainty.
In a nutshell, in the Pacejka model describing the tire/road interface, only the cornering stiffness K and the friction coefficient μ contribute significantly to the variation of the lateral force F_y .

2. References

Pacejka H.B., 2006: "Tyre and Vehicle Dynamics", Elsevier.

Cukier R.I., Levine R.I. and Shuler K.E., 1978: "Nonlinear sensitivity analysis of multiparameter model systems", Journal Computational Physics, 26, pp. 1-42.

Sobol I. M., 1993: "Sensitivity estimates for nonlinear mathematical models", Mathematical Modelling and Computational Experiments", 1, pp. 407-414.

Saltelli A., Tarantola S. and Chan K., 1999: " A quantitative model independent method for global sensitivity analysis of model output", Technometrics, 41, pp. 39-56

Saltelli A., Chan K. and Scott E.M., 2000: "Sensitivity Analysis", John Wiley and Sons, Chichester.

Frey H. C. and Patil S. R., 2002: "Identification and review of sensitivity analysis methods", Risk analysis, 22(3), pp. 553-578.

Saltelli A., 2002: "Sensitivity analysis for importance assessment", Risk Analysis, 22(3).

Saltelli A., Tarantola S., Campolongo F. and Ratto M., 2004: "Sensitivity analysis in practice", John Wiley and Sons, Chichester.