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Procedia Social and Behavioral Sciences 2 (2010) 7607–7608

Procedia
Social and Behavioral Sciences

Sixth International Conference on Sensitivity Analysis of Model Output

# Analytic Properties of High Dimensional Model Representations

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**Abstract**

The representation of multivariate functions is often given by means of functional ANOVA, in particular in Statistics, Simulation and Sensitivity Analysis. In despite of such use, its analytic and geometric properties have not been addressed yet. We derive conditions for continuity, differentiability, monotonicity and ultramodularity properties in functional ANOVA. We study the implications of these findings in multiattribute utility theory. We establish the conditions under which analytic properties of a multiattribute utility function imply the same properties of its one-attribute functions. The converse implications are also investigated.

"Keywords: Functional ANOVA; Multi-criteria Analysis, Multiattribute Utility Theory;"

A wide class of decision-making problems requires the quantitative assessment of multiattribute objective (utility) functions. The crucial point is the quantitative elicitation methods. [Fortemps et al (2008), Greco et al (2008)]. Practical advantages are given when two conditions occur. The first is when one can elicit one-attribute functions and next aggregate them. Additive and preferential independence are some of the assumptions that allow this procedure. (See the works of Keeney and Raiffa (1993), Baucells et al (2008), Fortemps et al (2008), Greco et al (2008); see also Angilella et al (2004) for non-additive utility function). Moreover, monotonicity and concavity properties on the single-attribute functions are useful in order to simplify numerical problem solution. The purpose of this work is to establish a theoretical framework to link the analytic and geometric properties of multi-attribute objective functions to the properties of their one-attribute components, without restrictions on the preference structure. In order to do it, we use the high dimensional representation model (HDRM), specifically in the version presented by Rabitz and Alis (1999)**.** Given a square integrable function $f$:$X\subseteq R^n \rightarrow R$ the following representation:

The paper has been submitted to an International Journal.

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doi:10.1016/j.sbspro.2010.05.140

$$\boldsymbol{f(x) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{i<j} f_{i,j}(x_i, x_j) + \cdots + f_{1,2,\ldots,n}(x_1, x_2, \ldots, x_n)} \qquad (1)$$

is called the functional ANOVA expansion of *f*.

We face the study of the analytic and geometric properties of functional ANOVA (or HDMR). Our first findings concern measurability, continuity and differentiability properties of *f*. All these properties are preserved in all the terms by the decomposition. Moreover we study the monotonicity and the ultramodularity properties [see Marinacci Montrucchio (2005) for a thorough description of the properties of ultramodular functions] of the terms in eq. (1). We prove that, if *f* is non-decreasing (non-increasing) then all first order terms are non-decreasing (non-increasing). Hence, first order terms have the same monotonicity properties of *f*. About the ultramodularity properties of functional ANOVA, results show that if *f* is ultramodular, then all first order terms in its functional ANOVA expansion are. For both monotonicity and ultramodularity, conditions for the higher order terms are provided.

In the theory of functional ANOVA, separable functions play a central role [see (Sobol' (2003)]. It is possible to prove that an additive function $h$ is monotonic if and only if each additive component $h_i$ is monotonic for all $i=1,2,...,n$. Similarly, if $h$ is additive then it is ultramodular if and only each additive component $h_i$ is ultramodular for all $i=1,2,...,n$. For multiplicative functions $g$, the same results hold provided that each component $g_i \geq 0,\ \forall i$. The above findings have direct implications in multiattribute decision making. In elicitation problems, separability is a crucial requirement to enable a quantitative assessment of preferences [Baucells et al (2008), Greco et al (2008)]. In particular we discuss the interpretation of these findings with respect to the relationship between preferences under certainty and under uncertainty [Keeney and Raiffa (1993)].

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