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Quasi-random-sampling high dimensional model representations for the construction of reduced discrete time state space dynamic models

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Abstract

In the context of real time model-based applications, complex high fidelity models may be computationally too expensive. Model order reduction and system identification techniques have been employed to transform complex models into equivalent reduced order models. However, most of the literature on model order reduction concerns linear time invariant dynamic systems, and the research into non linear model reduction is still on early stage. In this paper, we present a novel approach using quasi random sampling – high dimensional model representation (QRS-HDMR) to generate reduced discrete time state space dynamic models. The approach has the advantages of being able to handle the high dimensional case and produce affine discrete state space models, readily usable in control engineering applications.

Keywords: State Space Models, HDMR, Model Reduction, Linearization

1. Introduction

High dimensional Representations are an efficient technique to produce reduced models that can be orders of magnitude cheaper to evaluate. They were successfully applied to problems ranging from atmospheric chemistry modelling (Ziehn & Tomlin, 2008) to global uncertainty assessment (Li et. *al*, 2008). Even though they could generate equivalent models capable of capturing the dynamics of time-dependent systems (Li et. *al*, 2002), the technique was never applied to generate discrete time state-space dynamic models readily usable for control engineering purposes. We propose to apply HDMR techniques to the construction of reduced linear and quadratic state space models that can be used in a control engineering context.

2. Quasi-random sampling high dimensional representations

In this work we use the High Dimensional Model Representation (HDMR). HDMR uses an ANOVA type decomposition of the original functions:

$$\boldsymbol{f(x) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{1\le i<j\le n} f_{ij}(x_i\,, x_j) + \cdots + f_{12\ldots n}(x_1, x_2, \ldots, x_2)} \quad \textbf{(1)}$$

The expansion is hierarchically ordered in terms of the number of inputs involved in cooperative effects. Polynomial approximations are used to compute the value of component functions. Generally the coefficients of the polynomial

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HDMR. The improved RS-HDMR method based on QMC sampling (QRS–HDMR) was developed in Feil et al. (2009).

3. Procedure to generate state-space discrete time dynamic models

Usually, HDMR was used to generate fast equivalent operational models (FEOM) that can be used for simulation purposes. This approach consists of generating HDMR models for specific time points. However it is not possible to make use of such FEOMs in the context of control engineering, because of the necessity of using a receding time horizon. To overcome this issue we implement the following approach: Consider a general dynamic system of the form:

$$\dot{\boldsymbol{x}} = \dot{\boldsymbol{f}}(\boldsymbol{x}, \boldsymbol{u}) \tag{2}$$

Where x represents the vector of states of the model and u of controls inputs and f and g functions on the space of states and controls respectively. The algorithm relies on selection of target states for the model. For each of them, a QRS-HDMR model is obtained by sampling the state space; the input parameters are the initial values of states supplied to the ODE or DAE solver (and if any, the values of the control inputs. The output vector is the value of the states at time t+1. Finally it is possible to construct matrices defining the discrete linear time invariant system:

$$\boldsymbol{x(t+1)} = \boldsymbol{Ax(t)} + \boldsymbol{Bu(t)} + \boldsymbol{C} \tag{3}$$

Where $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ are matrices built from the coefficients in the first order HDMR expansion.

4. Conclusions and future work

The main advantages of this novel approaches are as follows:

- An automatic discretization and linearization method for continuous non linear models.
- The state-space models obtained are iterative and their construction is generally computationally cheap: Evaluation of original models from which the discrete time models are derived is carried out over very small time intervals, corresponding to the sample time of the desired model. For instance the construction a discrete time affine model from a three states nonlinear model, with 8192 sample points has a CPU of 120s.
- The method relies on high-dimensional model representations and is, in theory, applicable to large scale systems. Moreover, the global sensitivity indices derived from HDMR representations may be used for complexity reduction purposes as well as further analysis of the dynamic system (e.g. controllability).

Future work will deal with the application of the technique on systems of higher order as well as its mathematical formalization for control purposes.

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