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Theorems and Analytical Test Cases in Moment Independent Sensitivity Analysis

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Abstract

The lack of analytical test cases is one of the open issues in moment independent SA. In this work, by exploiting results in a previous paper by the authors, we present several analytical test cases. We also run corresponding numerical computations that corroborate the analysis.

"Keywords: Moment Independent Sensitivity Analysis; Global Sensitivity Analysis"

1. Introduction

Moment independent SA concerns appreciating the sensitivity of model output without reference to any of its particular moments. Early approaches to moment independent SA can be found in the works by Park and Ahn (1994) and Chun et al (2000). However, recent interest in moment independent SA has been raised by the δ importance measure [Borgonovo (2007), Borgonovo (2006), Liu and Homma (2009) and Borgonovo and Tarantola (2008)].

$$\delta_i = E_{X_i}\left[s_i(X_i)\right] \tag{1}$$

where

$$s_i(X_i) = \int \left| f_Y(y) - f_{Y|X_i}(y) \right| dy \tag{2}$$

In eq. (2), $s_i(X_i)$ measures the separation between the conditional and unconditional densities of the model output.

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Due to the recent introduction, however, several aspects of this importance measure still remain unexplored. By linking δ_i to the Minkowski-Hellinger's distance of order 1, Borgonovo et al (2010) prove two new properties of this importance measure: 1) invariance for monotonic transformation of the model output and 2)

$$\delta_i = E_{X_i}\left[P_Y(Y_+) - P_{Y|X_i}(Y_+)\right] \tag{3}$$

Eq. (3) states that δ_i equals the expected value of the probability that the model output belongs to Y_+, where Y_+ is the set of all y's is such that $f_{Y|X_i}(y) < f_Y(y)$.

These properties allow one to introduce a procedure for obtaining δ_i analytically. In particular, following the results in Borgonovo et al (2010), we illustrate δ_i in the following cases: additive model output with multivariate normally distributed model inputs (not necessarily independent), multiplicative model output with lognormally independent model inputs; and additive model output with uniformly distributed model inputs.

We discuss each test case analytically. We provide numerical results for each test case utilizing a Monte Carlo strategy based on Latin Hypercube sampling.

Our results pave the way to future studies in moment independent SA. The analytical test cases can be utilized to assess the efficacy of numerical estimation strategies (both concerning density approximation [Altomare et al, 2010] and the efficacy of sampling strategies [Castaings et al (2010)].

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