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Direct sensitivity computation for 1D hydrodynamic modelling *vs* classical empirical and Monte Carlo approaches

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Abstract

A finite volume method with a modified HLLC Riemann solver is proposed for the direct, equation-based, sensitivity solution of one-dimensional hyperbolic systems of conservation laws with discontinuous solutions. In the scope of hydrodynamic modelling, this method is applied to the shallow water equations and compared to the classical empirical method and the global approach using Monte Carlo simulations. Results obtained with the proposed approach show a better behaviour in the presence of shocks and a good reproduction of output distribution in only one simulation with possible variations of the input parameters up to 50%.
Keywords: direct sensitivity; hydrodynamics; discontinuous flow, uncertainty; local approach

1. Main text

Classical local sensitivity analysis methods meet problems when the model output becoms discontinuous, leading to locally infinite sensitivity values (Gunzburger, 1999). Global approaches, based on exploring the space of the input parameters, require a high number of simulations. A direct, equation-based, local method is proposed to deal with these two main drawbacks in the case of hydrodynamic models.

Governing equations: the one-dimensional shallow water equations can be written in vector form as

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{F}}{\partial x}=\mathbf{S} \qquad \text{with} \qquad \mathbf{U}=\begin{pmatrix} h \\ q \end{pmatrix}, \mathbf{F}=\begin{pmatrix} q \\ q^2/h+gh^2/2 \end{pmatrix}, \mathbf{S}=\begin{pmatrix} 0 \\ gh\ S_0-S_f \end{pmatrix} \tag{1}$$

where h is the water depth, q the unit-discharge, $S_0=-\partial z_b/\partial x$ the bottom slope (with z_b the bottom elevation) and S_f the friction slope. A sensitivity analysis consists in studying the influence of a small variation of a given parameter φ, on the solution of the flow equation. The sensitivity equations are obtained by differentiating (1) with respect to φ

$$\frac{\partial \mathbf{s}}{\partial t}+\frac{\partial \mathbf{G}}{\partial x}=\mathbf{Q} \qquad \text{with} \qquad \mathbf{s}=\begin{pmatrix} \eta \\ \theta \end{pmatrix}=\begin{pmatrix} \partial h/\partial\varphi \\ \partial q/\partial\varphi \end{pmatrix}, \mathbf{G}=\frac{\partial \mathbf{F}}{\partial \mathbf{U}}\mathbf{s}=\begin{pmatrix} \theta \\ c^2-u^2\ \ \eta+2u\theta \end{pmatrix}, \mathbf{Q}=\frac{\partial \mathbf{S}}{\partial \mathbf{U}}\mathbf{s}+\frac{\partial \mathbf{S}}{\partial \varphi}\varepsilon-\frac{\partial}{\partial x}\left(\frac{\partial \mathbf{F}}{\partial \varphi}\varepsilon\right) \tag{2}$$

where η and θ are the sensitivities of h and q with respect to φ; $c=\sqrt{gh}$ is the propagation speed of the pressure waves in the fluid at rest; u is the flow velocity defined as q/h; and ε is the so-called perturbation index, with ε=0 in

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the region where the parameter is to be left unchanged and ε=1 in the regions where φ is perturbed for the purpose of the sensitivity analysis. In the presence of discontinuous flow solutions (shocks such as hydraulic jumps, moving bores, etc.) differentiability is not guaranteed and the governing sensitivity equations must be modified so as to account for extra terms in the balance equation. In the case of the solution of the Riemann problem, with the left and right states denoted by subscript L and R, it can be shown (Guinot et al., 2009), that the complete jump relationships for the sensitivity are given by

$$\mathbf{s}_{\mathrm{L}} - \mathbf{s}_{\mathrm{R}} \quad c_s = \mathbf{G}_{\mathrm{L}} - \mathbf{G}_{\mathrm{R}} + \mathbf{R} \qquad \text{with} \qquad \mathbf{R} = \frac{\partial c_s}{\partial \varphi} \quad \mathbf{U}_{\mathrm{L}} - \mathbf{U}_R \tag{3}$$

This source term **R** being non zero only where **U** is discontinuous, (2) can be rewritten for the general case as

$$\frac{\partial \mathbf{s}}{\partial t} + \frac{\partial \mathbf{G}}{\partial x} = \mathbf{Q} + \mathbf{R}\delta_s \tag{4}$$

Numerical technique: the flow and sensitivity equations (1)-(2) are discretized using classical explicit finite volume formulation. A single Riemann problem is then defined for the hyperbolic part of the governing equations, that enables the fluxes computation using the HLL Riemann solver, with a modification for the sensitivity problem in case of shock. The two waves separating the intermediate region of constant state from the left and right states of the Riemann problem are assumed to be discontinuities, having respectively the following celerities: $\lambda^-;\lambda^+ \;=\; u-c;u+c$. The source terms **S** and **Q** in (1)-(2) are discretized using a classical upwinding procedure, and the contributions of an interface to its right and left cells are split into two parts with $-\lambda^-/(\lambda^+-\lambda^-)$ and $\lambda^+/(\lambda^+-\lambda^-)$ as respective weights. The point source term R is split into two contributions, using (3) and the two wave celerities:

$$\mathbf{R}^- = -\frac{\partial \lambda^-}{\partial \varphi} \quad \mathbf{U}_{\mathrm{L}} - \mathbf{U}_* \;\; ; \qquad \mathbf{R}^+ = -\frac{\partial \lambda^+}{\partial \varphi} \quad \mathbf{U}_* - \mathbf{U}_{\mathrm{R}} \tag{5}$$

Test cases: applying this method to the dam-break problem, with a sensitivity analysis with respect to the initial water depth in the dam h_0, allows for a correct location of the discontinuities in the sensitivity profiles. On the contrary, the empirical approach, that consists in performing two simulations using two slightly different values for h_0, shows a strong, artificial overshoot in the neighborhood of the shock.
Another comparative test case is performed for the backwater curve on a rectangular channel. Using a Monte Carlo approach, a high number of simulations are done to compute the water depth with gaussian distributions of the four parameters: Manning's friction coefficient n, bottom slope S_0, prescribed unit-discharge q and downstream boundary condition h_b. The deciles of the water depth distribution obtained as output are correctly reproduced using the direct sensitivity approach, that requires only one simulation. The mean square error between reconstituted and calculated water depth remains lower than 3% for variations up to 50% for all parameters except the friction coefficient for which the variation should remain lower than about 30% (using the following reference values: n=0.025m$^{-1/3}$s, q=1m^{2}s^{-1}, h_b=0.8m and S_0=10^{-3}).

Conclusion: a numerical technique is proposed to solve the one-dimensional shallow water sensitivity equations with possibly discontinuous solutions. Comparative test cases show that this method has a better behaviour near shocks than the empirical one and gives similar results as Monte Carlo approach, but in only one simulation, instead of several hundreds.

2. References

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