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Sensitivity analysis of a hierarchical qualitative model – the analysis of MASC

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Abstract

Sensitivity Analysis (SA) is applied to a hierarchical qualitative model built to assess the sustainability of cropping systems. Three approaches were tested to perform a first-order SA on such a model, assuming a fixed model structure and no correlation among input variables: (i) factorial designs combined with analysis of variance (ANOVA), (ii) conditional probabilities, (iii) Monte Carlo sampling (MC). If the complete factorial design is too large to be computed, MC and conditional probabilities represent efficient alternatives to perform an analysis of the overall qualitative model. Conditional probabilities exploit the hierarchical structure of the model to give exact first-order indices, while MC could be a more flexible approach for the introduction of correlations among variables. We discuss how such SA results can guide modellers and end-users in modelling and application phases.

Keywords: sensitivity index; ANOVA; Monte Carlo; conditional probabilities; hierarchical model; qualitative model; agricultural sustainability.

1. Main text

Cropping systems are complex systems that involve a lot of interactions between the biophysical, technical, social and economic components. Consequently integrative models are important tools to assess potential modifications. Performing global Sensitivity Analysis (SA) is therefore twofold: for the model-maker to understand how the model works and then to assess its quality; for the end-users to allow using the model taking into account its limits and uncertainties related to input variables and model parameters.

MASC (Multi-attribute Assessment of the Sustainability of Cropping systems, Sadok et al., 2009) is a multi-attribute hierarchical qualitative model developed to perform ex-ante multi-criteria analysis of the sustainability of cropping systems. The MASC model can be represented by the equation:

$$Z = f(L, \beta) \tag{1}$$

where Z is the model output (also called the root variable), L denotes the input variables (also called leaf variables), and ß denotes model parameters.

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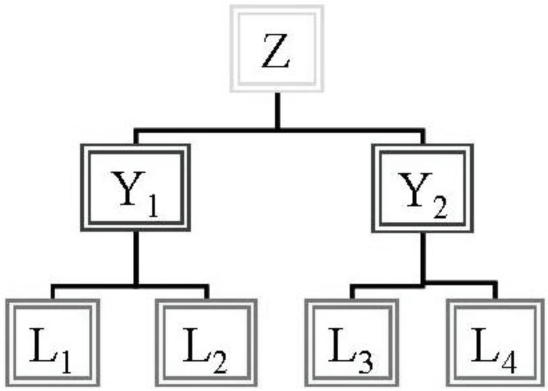
In the MASC model, the *f* function has a tree structure based on hierarchical relationships between qualitative variables: the leaf variables *L*, the intermediate (or aggregated) variables *Y*, and the root variable *Z*. Each aggregated variable, including *Z*, is defined by two or more direct descendants through a utility function (depending on ß), i.e. a table that represents the modality of *Y* associated with each combination of the descendants' modalities (Fig.1). Indirect descendants are all the descendants that belong to the same branch of the tree, but are not direct. The tree can be characterised by such quantities as the number of its branches, the depth in its tree structure or the number of modalities for each variable.



L_1	L_2	Y_1
high	high	high
high	medium	high
high	low	medium
medium	high	high
medium	medium	medium
medium	low	low
low	high	medium
low	medium	low
low	low	low

Figure 1 - Example of the model structure of MASC and of the utility function (ß) for Y_1.

When considering a fixed structure *f,* SA of the MASC model may be focused on the sensitivity of the root variable to the leaves; of the aggregated variables to their descendants, or of the root/aggregated variables to the variation of parameters (i.e. utility functions). The first two allow for a better understanding of the influences of the descendants on the aggregated variables, and are those analyzed here; the last kind of analysis considers how the parameters can affect the aggregated variables.

In the MASC model, the modalities of all variables are ordered. By considering them as a cardinal sequence from 1 to *n*, it is possible to calculate variance and expectation, and therefore Sensitivity Indices (SIs). In this paper, we focus on the first-order SI that measure the influence of the descendants *L* on an aggregated attribute, *Y*. They can be written as a measure of importance as proposed by Saltelli et al. (2000):

$$\mathrm{SI}(L;Y) = \frac{\mathrm{Var}(E(Y|L))}{\mathrm{Var}(Y)} \tag{2}$$

where $E(Y|L)$ denotes the expectation of *Y* conditional on *L*.

To perform such a SA on the MASC model three approaches were compared: (i) factorial designs, (ii) conditional probabilities, (iii) Monte-Carlo sampling.

Factorial designs are well adapted to qualitative input variables. A complete factorial design gives exact results for all factorial effects, but it could not be applied in our case due to the huge number (more than 3^{31}) of simulations required. Fractional factorial designs did not appear as a reliable alternative, because large interactions between factors were detected. Conditional probabilities were a very efficient alternative to calculate exact SI values. In this study, we developed a specific algorithm in R to perform these calculations for any hierarchical qualitative model such as MASC requiring around 3 seconds. MC approach gave similar results as conditional probabilities, and also required a short calculation time, about 12 seconds.

Results highlighted that the conditional probability approach better exploits the hierarchical structure of the model, while the MC approach seems more suitable if correlations among variables are accounted for. The practical interest of such SA results was then discussed with modellers and end-users in modelling and application phases.

2. References

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