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Development of a localized probabilistic sensitivity method to determine random variable regional importance

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Abstract

Knowledge of the critical region of an input pdf (tail, near-tail, central region) can provide valuable information towards understanding and improving a model through additional modeling or testing. A localized probabilistic sensitivity method was developed and demonstrated that will determine the region of a probability distribution that most affects the response. The methodology is based on discretizing the random variable pdfs using linear interpolation then sequentially injecting a localized disturbance into the pdf. A partial derivative of the probability-of-failure or the response moments (mean and standard deviation) with respect to each localized disturbance can be determined using sampling methods. The methodology can correctly locate the important region given sufficient samples.

Keywords: Probabilistic sensitivities, Score Function

1. Main text

A method to determine the relative importance of different regions of the input PDFs is developed as follows. The basic concept is to discretized each random variable PDF into regions, introduce a disturbance into each region, then determined the relative change in probability-of-failure or response moments (mean and standard deviation) for each regional disturbance. The concept is to linearize each CDF between discrete points, then input a local disturbance by perturbing the CDF value at the discrete points, one at a time. The response moments are then computed with the "perturbed" and unperturbed CDF's and the sensitivity estimated.

The sensitivity with respect to the disturbance can be determined using finite difference approximation as

$$\frac{\partial L_Z}{\partial F_i} \approx \frac{L'_Z - L_Z}{\Delta F_i} \tag{1}$$

where L'_Z represents a moment of the response, e.g., P_f, μ_Z, σ_Z, using the perturbed CFD, L_Z using the unperturbed CDF, and F_i represents the local cdf value. However, using finite difference method to estimate the sensitivity will be arduous and most likely very time consuming as a reanalysis is required for each region

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considered in every random variable. As such, a methodology based on the Score Function approach [Rubinstein and Shapiro, 1993] is developed such that the derivative can be estimated from a singe set of samples (Monte Carlo, LHS, or others).

Consider the derivative of the response output mean, μ_Z, with respect to a value F_i of X_i. The Score Function approach to computing the sensitivity is

$$\frac{\partial \mu_Z}{\partial \theta_i} = \frac{\partial}{\partial \theta_i}\int_{-\infty}^{\infty} Z(\mathbf{x}) f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} = \int_{-\infty}^{\infty} Z(\mathbf{x}) \left(\frac{\partial f_{X_i}(x_i)}{\partial \theta_i} \cdot \frac{1}{f_{X_i}(x_i)} \right) f_{\mathbf{x}}(\mathbf{x}) d\mathbf{x} =$$
$$\int_{-\infty}^{\infty} Z(\mathbf{x}) \kappa_{\theta_i}(\mathbf{x}) f_{\mathbf{x}}(\mathbf{x}) \cdot d\mathbf{x} = E[Z(\mathbf{x}) \kappa_{\theta_i}(\mathbf{x})] \tag{2}$$

The PDF for a linear CDF is

$$\begin{aligned} f_i &= \frac{F_i - F_{i-1}}{x_i - x_{i-1}} & x_{i-1} < x < x_i \\ f_{i+1} &= \frac{F_{i+1} - F_i}{x_{i+1} - x_i} & x_i < x < x_{i+1} \end{aligned} \tag{3}$$

and the resulting kernel function for the local disturbance is

$$\begin{aligned} \kappa_i &= \frac{1}{x_i - x_{i-1}} \frac{x_i - x_{i-1}}{F_i - F_{i-1}} = \frac{1}{F_i - F_{i-1}} & x_{i-1} < x < x_i \\ \kappa_{i+1} &= \frac{-1}{x_{i+1} - x_i} \frac{x_{i+1} - x_i}{F_{i+1} - F_i} = \frac{-1}{F_{i+1} - F_i} & x_i < x < x_{i+1} \\ 0 & & \text{otherwise} \end{aligned} \tag{4}$$

Thus, the kernel is a localized disturbance over the range $x_{i-1} < x < x_{i+1}$. Hence, the sensitivity of the mean with respect to the CDF value F_i can be estimated using sampling as

$$\frac{\partial \mu_Z}{\partial F_i} \approx \frac{1}{N} \sum_{j=1`}^{N} \left\{ \begin{matrix} Z(\mathbf{x}^j) \dfrac{1}{F_i - F_{i-1}} & x_{i-1} < x^j < x_i \\ Z(\mathbf{x}^j) \dfrac{-1}{F_{i+1} - F_i} & x_i < x^j < x_{i+1} \end{matrix} \right\} \tag{5}$$

where $\mathbf{x}^j$ denotes the jth realization of the random variables.

2. References

Rubinstein R.Y. and Shapiro, A. (1993) Discrete Event Systems, Sensitivity Analysis And Stochastic Optimization By The Score Function Method. J. Wiley & Sons, Chichester, England