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Model choice considerations and information integration using analytical hierarchy process

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Abstract

Using the theory of information-gap for decision-making under severe uncertainty, it has been shown that there exist irrevocable trade-offs between fidelity-to-data, robustness-to-uncertainty and confidence-in-prediction. The paper describes the assessment and trade-offs of these three components in a data-sparse application. To augment the data and corresponding modeling, a similar application with data and models is considered. A method of information integration is illustrated. Saaty's Analytic Hierarchy Process (AHP) is used to determine weights for two models and two experimental data sets, by forming all possible pair-wise comparisons between model output and experimental data.

Keywords: Inference; Inference Uncertainty; Information Integration; Analytical Hierarchy Process

1. Sensitivity analysis and model choice consideration

Sensitivity analysis is crucial in model construction, interpretation of model results, and model reduction. Many times, the basis for choosing between models is fidelity-to-data, or matching the physical measurements with accuracy. However, using the theory of information-gap for decision-making under severe uncertainty, it has been shown that there exist irrevocable trade-offs between fidelity-to-data, robustness-to-uncertainty and confidence-in-prediction. (This is a mathematical theorem, references [Ben-Haim 2001; Hemez and Ben Haim, 2004].) The main implication is that one cannot improve the ability of a simulation to match test data and, at the same time, make it more robust to modeling assumptions and/or improve prediction accuracy.

For example, a model that is purely calibrated to data may forego stages of validation and physical development. In such a case, sensitivity analyses based on the model should be restricted to very specific regimes, and uncertainty increases in situations that extend beyond the existing database used for calibration. We refer to “*calibration*” as one of the techniques for model development while “*validation*” refers to a process to assess the accuracy of a model and quantify prediction uncertainty, possibly, away from settings used for calibration. For assessing prediction accuracy, this scenario presents a distinction between the goals of

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sensitivity analyses and uncertainty quantification. This is because the analyst is likely to spend time questioning whether or not the model assumptions and data-production processes are well matched, and how the goal of validation could be demonstrated.

At the Los Alamos National Laboratory (LANL), a key component of science-based stockpile stewardship is an assessment of at least three components: 1) fidelity-to-data, or matching the physical measurements with accuracy; 2) robustness of predictions to lack-of-knowledge; and 3) assessing prediction accuracy and uncertainty in situations that extend beyond the existing database used for calibration. We desire predictions that capture the richness of physical experiments performed in diverse physical regimes, and models that are robust-to-ignorance while easily interpretable in terms of main effects. Increasing robustness, for example, by implementing first-principle physics-based models, usually comes at the price of reducing confidence-in-prediction; reduction in confidence usually occurs as goodness-of-fit decreases.

The paper describes the assessment and trade-offs of these three components in a data-sparse application. To augment the data and corresponding modeling, a similar application with data and models is considered. A method of information integration is illustrated. Saaty's Analytic Hierarchy Process (AHP) [Saaty, 1980] is used to determine weights for two models and two experimental data sets, by forming all possible pair-wise comparisons between model output and experimental data, corresponding to the six pair-wise combinations of information in four boxes.

The six pair-wise comparisons form the upper triangle of a matrix, with values equal to one on the diagonal and inverse scores in the lower triangle. An eigenvalue decomposition of matrix AHP is performed next to obtain weights for each of the four boxes in Figure 1. The decomposition is defined simply as:

$$AHP = \sum_{k=1 \dots 4} \lambda_k \cdot \phi_k \cdot \phi_k^T, \quad (1)$$

where the eigenvalue λ_k can be interpreted as an “influence factor” of the k^{th} mode of the decomposition, and Φ_k is the corresponding eigenvector. The first (principal or dominant) eigenvalue represents a majority of the information content of matrix AHP . The corresponding eigenvector Φ_1 has four components whose values can be normalized to sum to one, and serve as weights for two model outputs and two experimental databases. We will also demonstrate the D_N metric [Booker et al., 2006] to measure fidelity-to-data because it can handle multiple diagnostics while accounting for experimental variability:

$$D_N \equiv \sum_{k=1 \dots N} \left(\frac{y_k^{Test} - y_k(p)}{\sigma_k^{Test}} \right)^2, \quad (2)$$

where σ^{Test} denotes the variability and index “k” sums over the diagnostics.

2. References

- Ben-Haim, Y., 2001 “Robust Rationality and Decisions Under Severe Uncertainty,” *Journal of the Franklin Institute*, Vol. 337, 2000, pp. 171-199; Ben-Haim, Y., **Information-Gap Decision Theory: Decisions Under Severe Uncertainty**, Academic Press, 2001.
- Hemez, F.M., Ben-Haim, Y., “The Good, the Bad and the Ugly of Predictive Science,” *4th International Conference on Sensitivity of Model Output*, March 8-11, 2004, Santa Fe, NM.
- Saaty, T.L., 1980, “*Analytic Hierarchy Process: Planning, Priority Setting, and Resource Allocation*,” New York: McGraw-Hill, 1980.
- Booker, J.M., Langenbrunner, J.R., Hench, K.W., Martz, H.F., “Interpretations of Langenbrunner’s D_N Metric for V&V,” Los Alamos National Laboratory Technical Report, May, 2006, LA-UR-06-3720.