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Investigation on the Structure of a Networked System

S. La Rovere[a], P.Vestrucci[b]

[a]NIER Ingegneria, Altabella 3, Bologna 40127, Italy
[b] Facoltà di Ingegneria – DIENCA, Viale Risorgimento 2, Bologna 40136, Italy

Abstract

We present a general approach to investigate the Structure of a Networked System by ranking its elements through additive, first order, Importance and Sensitivity measures. The performances of the System are defined in term of Partial and Global Risks, which are referred to each User node and to the whole Network respectively. The ranking of the Edges (subjected to failure and repair events) by means of additive measures allows to rank the (unfaultable) User nodes. The System's Structure is investigated with reference to the whole range [0,1] of the input variables. The interactions among them are investigated by analyzing the trend of the Total order importance measure for different values of the finite change of the input variables.
Keywords: Networked System, Structure, Importance, Sensitivity

1. Main text

We present a general approach to assess the Structure of a Networked System, made up of unfaultable "Source" and "User" nodes, joined by (N) directed Edges which are subjected to failure and repair events. A weight (w_i) is assigned to each (i) User node, as the damage produced when it is unavailable (i.e. not connected to a Source node, by means of a path made by available Edges). A weight (w_s) results for each System's status, as the whole damage caused by all the unavailable User nodes. The System's performances are defined in term of Partial and Global Risks, which are referred to each User node and to the whole Network respectively. The Global Risk is the sum over the System's states of the product between w_s and the probability that the System is in that status or the sum over the User nodes of the product between w_i and their unavailability (La Rovere et al., 2009).

The Structure of a Networked System is investigated by ranking its elements through the first order Differential Importance Measure (DIM) (Borgonovo et al., 2001) and a Hybrid measure (Saltelli et al., 2004). Both the measures can be evaluated on the basis of the Jacobian matrix of the System (i.e. the first order partial derivatives of the Partial Risks, with respect to the unavailability of the Edges). Their comparison clarifies the different meanings of an Importance and a Sensitivity ranking of elements: the first one is referred to the contribution on the output variable of each input variable; the second one is referred to the contribution on the uncertainty associated with the output variable of the uncertainty associated with each input variable. The adoption of additive measures for the ranking of the Edges allows the ranking of the (unfaultable) User nodes; this approach allows to overcome the ranking of the nodes by means of their "Degree" (number of related Edges) (Pride et al., 2009).

The DIM (under the hypothesis of uniform change of the unavailability of Edges) is referred to the Global Risk; in this case, the measure is additive with respect to a set of Edges and to a set of User nodes (La Rovere et al., 2009).

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The Hybrid Measure is defined according to the Decomposition of the variance associated to the Global Risk, on the basis of its first order approximation. It includes the contributions due to the correlations among the Partial Risks; indeed, starting from the independent failures and repairs of Edges, the Structure of the Network introduces correlations among the states of its User nodes, and therefore among the Partial Risks (La Rovere et al., 2010). The Hybrid Measure for the Edge j with respect to the Partial Risk i provides the fraction of the variance associated with the Global Risk due to the Edge j, by means of the User node i. The Hybrid Measure can be also estimated by means of Sampling techniques, through N OAT simulations. If the variance associated with the Global Risk is estimated by a further simulation, during which the unavailability of all Edges are variable contemporaneously, the Hybrid Measure coincides with the Main sensitivity index (Sobol, 1993).

The analysis of the Structure of a Networked System is developed under the following assumptions:

- the failure and repair of the Edges are independent; a common value is assumed for their unavailability;
- the whole range [0,1] of the input variable is considered; this approach is more general than the traditional one proposed for the analysis of the Structure of a System by Birnbaum (1969) and already applied in previous works for the analysis of the Structure of a Networked System (La Rovere et al., 2009);
- fictitious uncertainties are assigned to the unavailability of the Edges; they are represented by a common probability distribution (Normal distribution with a small enough standard deviation);
- a common value $w_i = 1$ is taken on for the weights assigned to the User nodes.

Under the above assumptions, the DIM and the Hybrid Measure for the Edge j with respect to the Partial risk i are defined by the following equations (the general expressions for $w_i \neq 1$ are not provided in the abstract):

$$DIM_j^i = \frac{\dfrac{\partial U^i}{\partial U_j}}{\sum_{i=1}^{Nn}\sum_{j=1}^{Nc}\dfrac{\partial U^i}{\partial U_j}} \qquad I_j^i = \frac{{}_j^i Var[R]}{Var[R]} = \frac{\dfrac{\partial U^i}{\partial U_j}\cdot\left(\sum_{l=1}^{Nn}\dfrac{\partial U^l}{\partial U_j}\right)}{\sum_{i=1}^{Nn}\sum_{j=1}^{Nc}\left(\dfrac{\partial U^i}{\partial U_j}\cdot\sum_{l=1}^{Nn}\dfrac{\partial U^l}{\partial U_j}\right)} \tag{1}$$

The DIM for the Edge j with respect to Global Risk is the sum of the measures over the Partial Risks; the ranking of the User nodes is defined by the sum of the measures over the Edges. The same is for the Hybrid Measure.

The interactions among the unavailability of Edges due to the Structure of the Networked System are analyzed by means of the Total order importance measure (Borgonovo, 2010). The Measures for the N Edges of the Network with respect to the Partial Risks are estimated by means of N+2 evaluation of the model, for a finite change of their unavailability. The Total order importance measure with respect to the Global Risk is the sum of the measures over the Partial Risks. The Structure of the Networked System is investigated on the basis of the trend of the Total order importance measure for different values of the finite change in the whole range of the input variables [0,1].

Analytical evidences and an applicative case are provided.

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