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Sensitivity analysis for quantile regression

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Abstract

In this paper, we explore sensitivity analysis for quantile regression and confront it with diagnostic testing. Every model is misspecified (in the sense that no model coincides with the data-generating process), but a model is useful if the parameters of interest (the focus) are not sensitive to small perturbations of the underlying assumptions. Magnus and Vasnev (2007) found that in the case of mean regression (and more generally in a maximum likelihood framework) both, sensitivity and diagnostic, are important and often (asymptotically) independent. One expects similar result for quantile regression as well. However, the relationship between sensitivity and diagnostic varies for different quantiles. We introduce a sensitivity statistic for quantile regression, compare it with the mean regression sensitivity and look at its performance in simulations.

Keywords: Sensitivity analysis; Quantile regression; Diagnostic test

1. Motivation and main contribution.

The main difference between the case analyzed by Magnus and Vasnev (2007) and quantile regression is that the objective function of the latter is not continuously differentiable, so the simple closed form solution does not exist. However the sensitivity question is still valid and one is interested in the effect of additional parameters in the model on the main parameter of interest.

We follow the model presented by Koenker (2005) and look at the conditional quantile function

$$Q_y(\tau \mid x) = x'\beta(\tau), \tag{1}$$

which can be consistently estimated by minimizing

$$\sum \rho_\tau(y_i - x_i'\beta(\tau)), \tag{2}$$

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where $\rho_\tau(u) = u(\tau - I(u < 0))$.

Often there are other parameters in the models that are not of the main interest, but that represent an important characteristic of the modeled data. In this case the conditional quantile function will depend on the vector of other parameters, θ. This dependence $\beta(\tau,\theta)$ is interesting to look at.

The local sensitivity can be naturally derived from the Taylor expansion

$$\beta(\tau,\theta) = \beta(\tau,\theta_0) + s(\tau,\theta_0)(\theta - \theta_0) + r,, \tag{3}$$

where the sensitivity statistic, $s(\tau,\theta_0)$, is the first derivative of $\beta(\tau,\theta)$ with respect to θ at the point of the true parameter value θ_0 and r is the remaining term.

As mentioned before, the objective function is not continuously differentiable. However, the problem is well known in the area of mathematical programming and Castillo et al (2004) provide the framework for addressing a similar problem for the least absolute deviation estimator, which is a special case of quantile regression. Most of the sensitivities can be obtained directly after solving the problem for $\beta(\tau,\theta_0)$ and finding the dual variables of the solution.

The performance of $s(\tau,\theta_0)$ is investigated in simulations with three core examples: misspecification in the mean, misspecification in the variance and misspecification in the distribution. Its relationship with the Wald test and the quantile likelihood ratio test is examined as well. A simulation with 10 000 repetitions is used to investigate the properties where analytical solutions are not available.

The fact that $s(\tau,\theta_0)$ depends on τ is, of course, not surprising, but the character of this dependency is interesting. For τ close to the center of the distribution the sensitivity is low, it is increasing when we move towards the tails. This fact confirms robustness of the median regression and, at the same time, cautions us about the reliability of tail estimation.

2. References

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