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Robust Method for Sensitivity Analysis in Simulation Model/a Comparison Study

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Abstract

Kleijnen proposed using Ordinary Least Squares method combining with experimental design to estimate polynomial regression metamodels, but I/O data violates some classical assumptions of OLS as the correlation between output which due to common random numbers and Heterogeneous variances which caused by using different factor combinations. Thus Kleijnen and David referred to using repeated OLS (OLSR) or Generalized Least Squares (GLS) as a robust methods instead of OLS.

In this study we compare these two methods using a simulation model M/M/1 which represented by a Queuing model in the repair and maintenance fields. We validated the estimated first order polynomial regression meta-model using adjusted R2 and Relative average absolute error, Our results demonstrate that As a result, OLSR is more efficient and more validation than GLS method.

Keywords: Sensitivity analysis ; regression metamodel ; least squares ; robust method ; validation

1. Main text

The sensitivity analysis of simulation models is considered one of the most important analyses for the development of simulation models to know the most important input variables.

Kleijnen proposed using statistical theory on design of experiments in combination with estimated polynomial regression meta-modeling to getting highly efficient estimations and non overlapping effects Moreover, the design of simulation experiments allows for the estimation of interactions, i.e. high order effects.

Let $X = (x_{ij})$, ($i = 1,\ldots, n$; $j = 1,\ldots, q$) denotes as matrix of explanatory variables , where n denotes to the number of factor combinations in that experiment, using stochastic simulation by repeating i^{th} factor combination m_i times ($m_i > 1$, $i = 1,2,\ldots,n$) and let $\underline{w}$ be a vector of N ($N = \sum_{i=1}^{n} m_i$) of simulation outputs. Then Ordinary Least Squares method (OLS) can be used to estimate Sixth International Conference on Sensitivity Analysis of Model Output regression metamodels, but I/O data violates some classic assumptions of OLS:

1-Cov (wi, wj) $\neq 0$, $i \neq j$ which caused by using most of simulation runs CRN

2- Heterogeneous variances because of using different factor combinations.[3]

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As a result, errors are correlated and not have equal variances. Then, $\hat{\beta}$ not a best linear unbiased estimator (BLUE), thus Kleijnen, David [4] denoted to using repeated OLS (OLSR) or Generalized Least Squares, GLS (Aitken) estimator which extends the Gauss–Markov theorem to the case where the error vector has a non-scalar covariance matrix – the GLS estimator is a BLUE.

In this study we compare these two methods using a simulation model represented by a Queuing model in the repair and maintenance fields with one server, and an arrival rate (λ) and a service rate (μ) which is called M/M/1 model.Transform the relation between $\hat{\lambda}, \overline{w}$ and $\hat{\mu}, \overline{w}$ into linear relation by using semi-logarithmic transformation. On the other hand, Kleijnen (1987) proves that using responses average $\overline{w}_i$ instead of single outputs is sufficient to estimate unknown parameters using OLS method.

Firstly, we estimate a first order polynomial regression metamodel using 2^2 design by simulate M/M/1 model repeating each factor combination 1000 times to get w1,w2,…,w1000 , then compute $\ln(\overline{w}_i)$ as follows:

$$\ln(\overline{w}_i) = \ln\left(\sum_{k=1}^{1000} w_k / 1000\right) \qquad , \qquad i = 1,2,3,4 \tag{1}$$

Repeat that 30 times (m=30). As we shown previously, OLS is not robust, so, we'll estimate $\hat{\beta}$ according to:

1-OLSR method

$$\hat{\beta}_j = \frac{\sum_{i=1}^{30} \hat{\beta}_{ij}}{30} \qquad , \qquad j = 0,1,2 \tag{2}$$

With variances:

$$\operatorname{var}(\hat{\beta}_j) = \frac{\sum_{r=1}^{30} (\hat{\beta}_{j,r} - \overline{\hat{\beta}}_j)^2}{29} \qquad , \qquad j = 0,1,2 \tag{3}$$

We validated the estimated first order polynomial regression meta-model using:

i. adjusted R^2 and reach to : $R^2_{adj} = 0.90$
ii. Relative average absolute error (RAAE) as follows:[1]

$$RAAE = \frac{\sum_{i=1}^{n} \left|\overline{w}_i - \hat{y}_i\right|}{n * STD} \tag{4}$$

Where STD standard deviation of error. The smaller the value of RAAE, the more accurate the metamodel.
We found that RAAE=0.866 and All effects where significant at 5% level (using tabulated t with (m-1) d.f instead of n-q). [3]

2-GLS method

$$\underline{\tilde{\beta}} = (X'\hat{S}^{-1}_{\overline{w}}X)^{-1} X'\hat{S}^{-1}_{\overline{w}} \underline{\overline{w}} \tag{5}$$

Where:

$$\hat{S}_{i',i} = \sum_{r=1}^{m} (w_{i',r} - \overline{w}_{i'})(w_{i,r} - \overline{w}_i) / m(m-1) \qquad , \qquad i', i = 1, \ldots, n \tag{6}$$

After estimate first order polynomial effects we reach to:
$R^2_{adj} = 0.77$, RAAE= 1.303
As a result, OLSR is more efficient and more validation than GLS method.

2. References

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